Bernards Heath Infant and Nursery School

Bernards Heath Schools • United in Learning •

Calculation Policy

March 2017

To be read in conjunction with our Maths Policy

Written Calculations Policy

Introduction

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the National Curriculum and EYFS as appropriate. Through the policy, we aim to link key manipulatives and representations in order that the children become fluent through each strand of calculation. This policy ensures consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. Through differentiated challenge children move at the pace appropriate to them, teachers present strategies and equipment to enhance the children's level of understanding. However, we would expect the majority of each class to be working at age-related expectations.

Age Related Expectations

By the end of FS2 children should: Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.
By the end of Y1 children should: Begin to establish the relationship between addition and subtraction – 2 + 5 = 7 and 7 – 2 = 5 Realise the effect of adding and subtracting zero Represent and use number bonds and related subtraction facts within 20 Add and subtract 1 and 2 digit numbers to 20, including 0 Read, write and interpret mathematical statements involving addition, subtraction and equals signs (+, -, =) Solve 1 step problems that involve addition and subtraction using concrete objects and pictorial representations and missing number problems such as 7 = ? – 9 Solve 1 step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays, with support from the teacher Find half and quarter of an object, shape or quantity – discrete quantities (e.g. cherries on a plate) and continuous quantities (e.g. water) Recognise and create repeating patterns with objects and shapes Create equivalent expressions (2 + 5 = 5 + 2)
By the end of Y2 children should: Recognise and use the inverse relationship between addition and subtraction and uses this to check calculations and missing number problems Show that addition of two numbers can be done in any order and subtraction of one number from another cannot Show that multiplication of two numbers can be done in any order and division of one number by another cannot Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 *such as 3+7=10, 10-7=3 and 7=10-3 to calculate 30+70=100, 100-70=30 and 70= 100-30* Add and subtract numbers, using concrete objects and pictorial representations, mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers Recalls and uses multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers e.g. *pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition* Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs **Continued…**

Find fractions of a length, shape, set of objects or quantity
Write simple fractions *e.g. ½ of 6 = 3*
Recognise the equivalence of 2 quarters (2/4) and half ½
Solve problems with addition and subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts
Recognise patterns within the number system up to and beyond 100

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school:

To add and subtract successfully, children should be able to:
· Recall all addition pairs of single digit numbers up to 9+9 (e.g. 7 = 2 + 5, 6 +1, 3 + 4)
· Recall number bonds to 10
· Recognise addition and subtraction as inverse operations
· Add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
· Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
· Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
· Use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:
· Add and subtract accurately and efficiently
· Recall all multiplication and division facts within the 2, 3, 4, 5, and 10 times tables
· Use multiplication and division facts to estimate how many times one number divides into another etc.
· Know the outcome of multiplying by 0 and by 1 and of dividing by 1
· Understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
· Derive other results from multiplication and division facts, using what you already know to work out something new
· Notice and recall inverse facts with increasing fluency
· Partition numbers into 100s, 10s, 1s and other multiples
· Understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
· Understand the effects of scaling by whole numbers and fractions
· Investigate and learn rules for divisibility

Progression in Addition and Subtraction

Addition and subtraction are connected.

Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

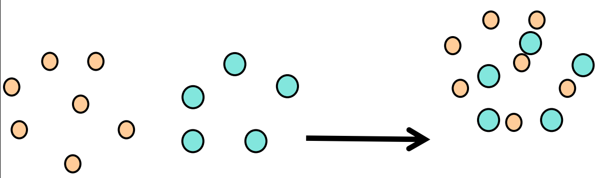
Addition is taught as the inverse of subtraction (and vice versa) and concepts are taught alongside one another across the school e.g. 5+3, 3+5, 8-5, 8-3.

ADDITION

Combining two sets (aggregation)

Putting together – two or more amounts or numbers are put together to make a total

7 + 5 = 12



Count one set, then the other set. Combine the sets and count again. Starting at 1.

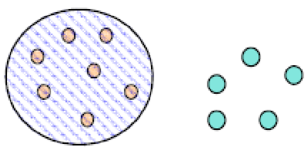
Counting along the bead string, count out the 2 sets, then draw them together, count again. Starting at 1.

Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting

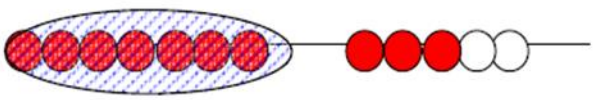
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

Bead strings:



Make a set of 7 and a set of 5. Then count on from 7.

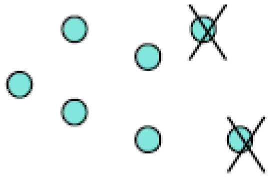
Multi-link Towers:

SUBTRACTION

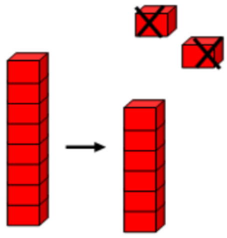
Taking away (separation model)

Where one quantity is taken away from another to calculate what is left.

8 – 2 = 6



Multilink towers- to physically take away objects.

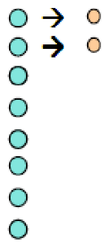


Finding the difference (comparison model)

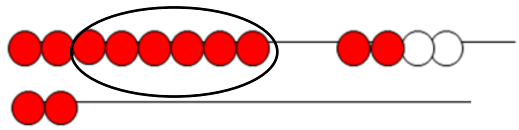
Two quantities are compared to find the difference.

8 – 2 = 6

Counters:

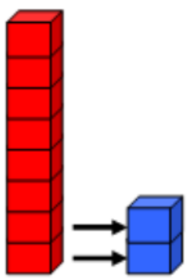


Bead strings:



Make a set of 8 and a set of 2. Then count the gap.

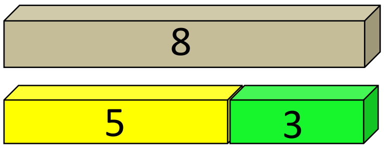
Multi-link Towers:



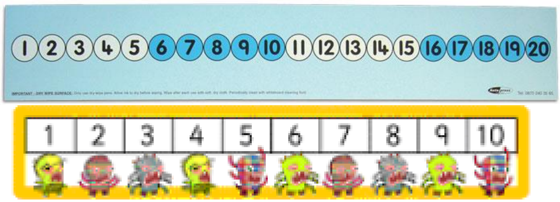
Numicon:

7 + 3 =

Cuisenaire Rods:



Number tracks:

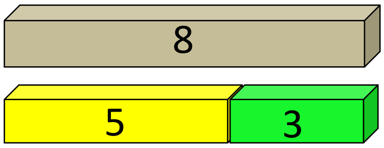


Start on 5 then count on 3 more

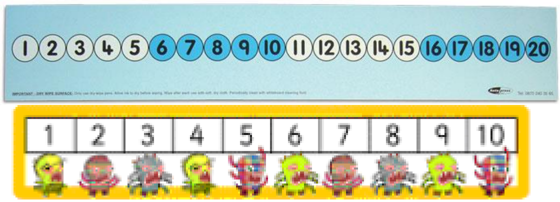
Numicon:

8 – 2 =

Cuisenaire Rods:



Number tracks:



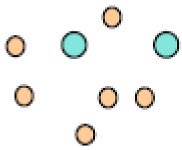
Start with the smaller number and count the gap to the larger number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.

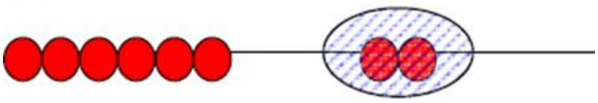
8 – 2 = 6

Counters:

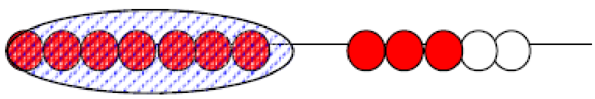


Bead strings:

8 – 2 = 6



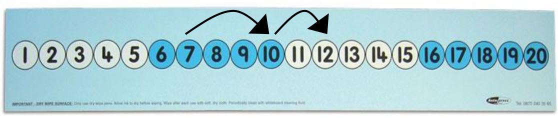
Bead string:



7 + 5 is decomposed/partitioned into 7 + 3 + 2.

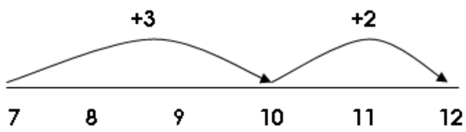
The bead sting illustrates 'how many more to the next multiple of 10?' (the children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:

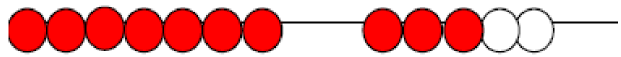


Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line



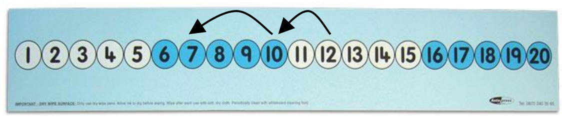
Bead string:



12 – 7 is decomposed/partitioned in 12 – 2 – 5.

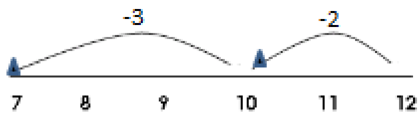
The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:



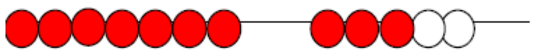
Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



Counting up or 'Shop keepers' method

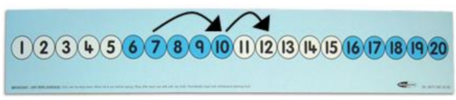
Bead string:



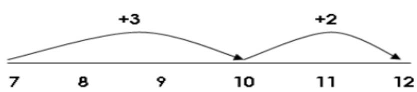
12 – 7 becomes 7 + 3 + 2.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:



Number Line:

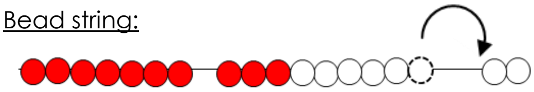


Compensation model (adding 9 and 11)

This model of calculation encourages efficiency and application of known facts (how to add ten)

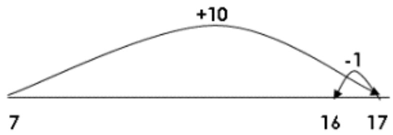
7 + 9

Bead string:



Children find 7, then add on 10 and then adjust by removing 1.

Number line:



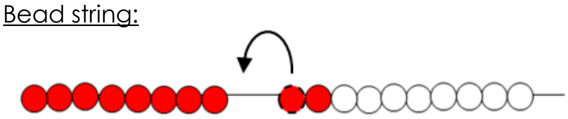
100 Square:

7 + 9

Jump down to add 10 then subtract 1.

18 – 9

Bead string:



Children find 18, then subtract 10 and then adjust by removing 1.

Number line:

100 Square:

18 - 9

Jump up to subtract 10 then add 1.

Working with larger numbers

TO + TO

Ensure that the children have been transitioned onto Diennes and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

34 + 23

Diennes:

Children create the two sets with Diennes and then combine; ones with ones, tens with tens.

Although the calculation below shows the process of the aggregation model, children would not be expected to record this way at this stage:

34 + 23

4 + 3 = 7

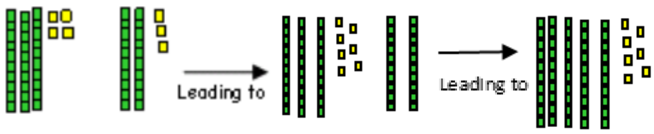
30 + 20 = 50

50 + 7 = 57

Partitioning (Augmentation model)

Diennes:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



At this stage, children are taught to record the calculation as below:

34 + 23

34 + 3 = 37

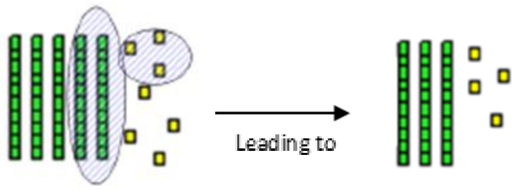
37 + 20 = 57

Number line:

Take away (Separation model)

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

57 – 23 = 34



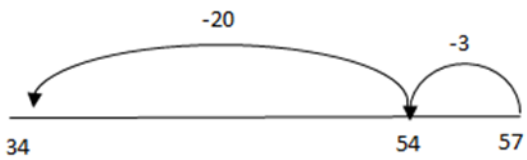
At this stage, children are taught to record the calculation as below:

57 – 23

57 – 3 = 54

54 – 20 = 34

Number Line:

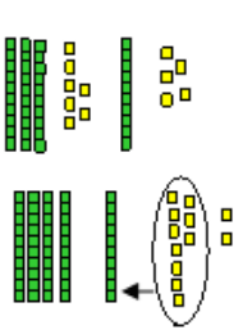


Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the units are greater than 10? Introduce the term 'exchange'. Using the Diennes equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Diennes:

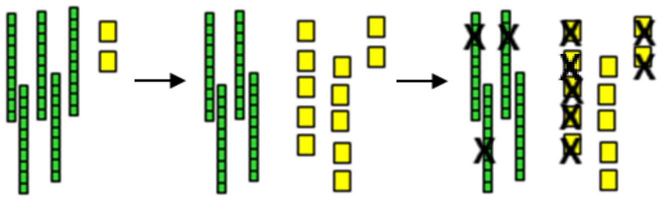
37 + 15



Discuss counting on from the larger number irrespective of the order of the calculation.

Diennes:

52 – 37 = 15



Gradation of difficulty- addition

1. No exchange
2. Extra digit in the answer
3. Exchanging ones to tens
4. Exchanging tens to hundreds
5. Exchanging ones to tens and tens to hundreds
6. More than two numbers in calculation
7. As 6 but with different number of digits
8. Decimals up to 2 decimal places (same number of decimal places)
9. Add two or more decimals with a range of decimal places.

Gradation of difficulty- subtraction

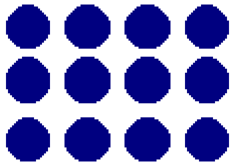
1. No exchange
2. Fewer digits in the answer
3. Exchanging tens for ones
4. Exchanging hundreds for tens
5. Exchanging hundreds to tens and tens to ones
7. As 6 but with different number of digits
8. Decimals up to 2 decimal places (same number of decimal places)
9. Subtract two or more decimals with a range of decimal places.

<u>Progression in Multiplication and Division</u>

Multiplication and division are connected.

Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The array above, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12,

4 x 3 =12,

3 + 3 + 3 + 3 = 12,

4 + 4 + 4 =12.

And it is also a model for division

12÷4 = 3

12÷3 = 4

12 – 4 – 4 – 4 = 0

12 – 3 – 3 – 3 – 3 = 0

Multiplication is taught as the inverse of division (and vice versa) and concepts are taught alongside one another across the school.

MULTIPLICATION	DIVISION

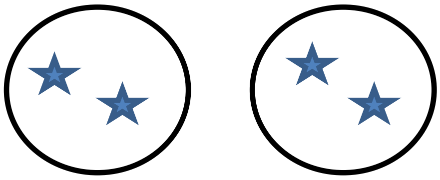
Early experiences

Multiplication

Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.

Division

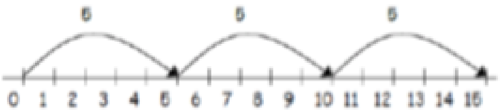
Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.



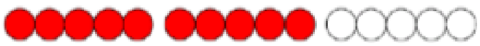
Repeated addition (repeated aggregation)

3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3

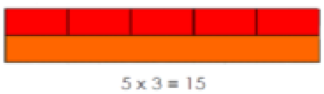
Children learn that repeated addition can be shown on a number line.



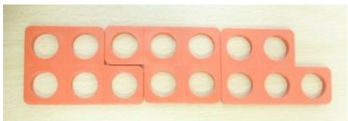
Children learn that repeated addition can be shown on a bead string.



Children also learn to partition totals into equal trains using Cuisenaire Rods



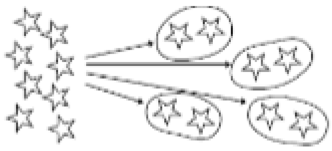
Children learn to show repeated addition using Numicon



5 + 5 + 5 = 15

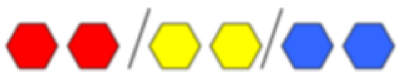
Sharing equally

6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.



Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?



Scaling

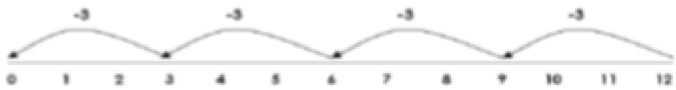
This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount.

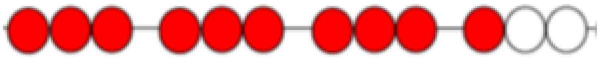
For example, find a ribbon that is 4 times as long as the blue ribbon.

Or build a tower that is half as high and twice as wide.

Repeated subtraction using a bead string or number line

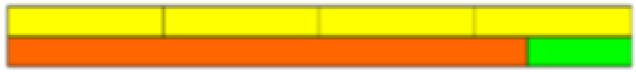
12 ÷ 3 = 4





The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

Cuisenaire Rods also help children to interpret division calculations.



Grouping involving remainders

Children move onto calculations involving remainders.

13 ÷ 4 = 3 r1

Or using a bead string see above.

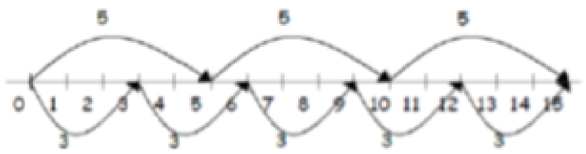
Commutativity

Children learn that 3 x 5 has the same total as 5 x 3.

This can also be shown on the number line.

3 x 5 = 15

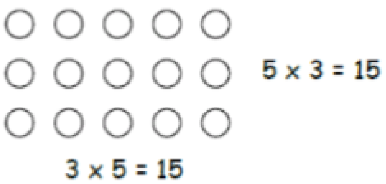
5 x 3 = 15



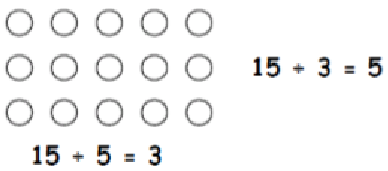
Children learn that division is **not** commutative and link this to subtraction.

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.



Inverse operations

Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

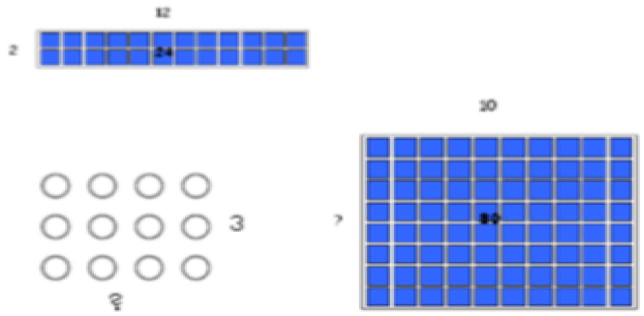
12 ÷ 4 = 3

Children use symbols (such as a box or a question mark) to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20 3 x □ = 18 ? x □ = 32

24 ÷ 2 = □ 15 ÷ ? = 3 □ ÷ 10 = 8

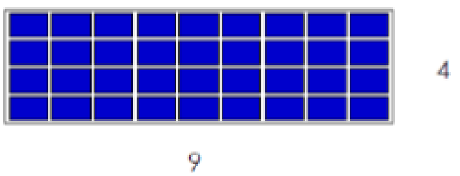
This can also be supported using arrays: e.g. 3 X ? = 12



Partitioning for multiplication

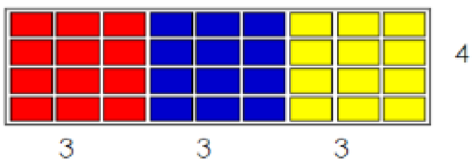
Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

9 x 4 = 36



Children should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks.

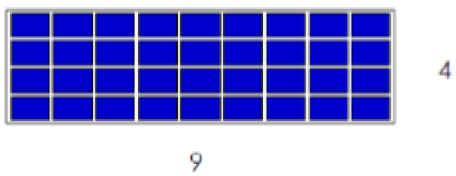
9 x 4 =



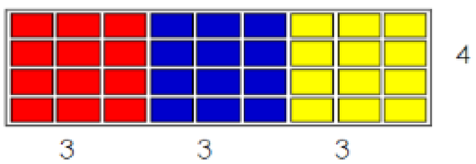
Partitioning for division

The array is also a flexible model for division of larger numbers

36 ÷ 4 = 9



Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.



3 x 4 = 12

12 + 12 + 12 = 36

Gradation of difficulty (Short multiplication)

1. TO x O no exchange
2. TO x O extra digit in the answer
3. TO x O with exchange of ones into tens
4. HTO x O no exchange
5. HTO x O with exchange of ones into tens
6. HTO x O with exchange of tens into hundreds
7. HTO x O with exchange of ones into tens and tens into hundreds
8. As 4-7 but with greater number digits x O

Gradation of difficulty (Short division)

1. TO ÷ O no exchange no remainder
2. TO ÷ O no exchange with remainder
3. TO ÷ O with exchange no remainder
4. TO ÷ O with exchange, with remainder
5. Zeroes in the quotient e.g. 816 ÷ 4 = **204**
6. As 1-5 HTO ÷ O
7. As 1-5 greater number of digits ÷ O
8. As 1-5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3
9. Where the divisor is a two digit number

9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O	See below for gradation of difficulty with remainders
	<u>Dealing with remainders</u> Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly. e.g. · I have 62p. How many 8p sweets can I buy? · Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed? **<u>Gradation of difficulty for expressing remainders</u>** 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal